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ON THE MEASUREMENT OF WATER BY A SMALL
VENTURI METER.

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INTRODUCTION.

The problem of accurately measuring the continuous flow of water is one that has received a large amount of attention, and many forms of meters have been devised for attaining this object. A form of meter, which has come into general use within the last few years, is that known as the Venturi meter.

This form of meter is especially applicable to water-mains, as the head necessary to operate the meter is small and the quantities may be extremely large.

Herschel (1) has shown that, in meters applicable to mains of 1 foot and 9 feet diameter respectively, the flow is proportional to the square root of the difference of head between the two sections of the cone forming the essential part of the meter, and this holds through the whole range.

In the present paper the meter used was a very small one, and the principal object of the experiments was to determine the range of the law

$$\text{quantity} = a \sqrt{H}$$

and, as a consequence, the variation of the "constant" in the equation connecting the flow with the dimensions of the meter and the head.

The meter used was especially designed and constructed for the experiments, and had several advantages over the ordinary form.

(1.) The Venturi water meter. Trans. Am. Soc. C.E., 1887.

DESCRIPTION OF APPARATUS.

In its simplest and more usual form, the Venturi meter consists of two frustra connected at their smaller ends by a short cylindrical tube or "throat," their outer ends being fitted to the pipe through which the water it is desired to measure is passing, and provided with means for attaching pressure gauges at the throat and at the outer ends of the cones.

At the points where it is desired to measure the pressure in the meter a pressure-chamber encircles the pipe and communicates with the interior by a number of small holes drilled as nearly radially as possible. It is essential for accuracy that the holes should be truly radial and the edges without burr, the attainment of which is a matter of considerable difficulty, especially in a small meter.

The meter used in these experiments overcomes the objections mentioned above.

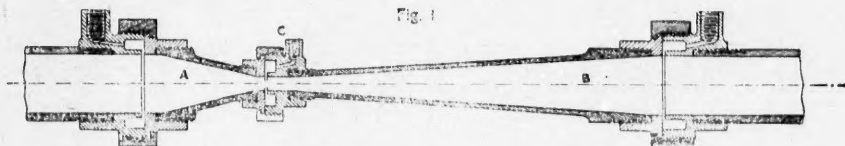
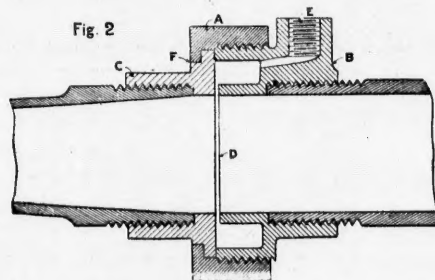


Fig. 1 is a longitudinal section of the meter consisting of a short "up stream" cone, A, and a "down stream" cone, B, connected by a combined coupling and pressure chamber, C, which forms the throat.

The Venturi is connected to the pipe by two other couplings of the same form. The combined pressure chamber and coupling is the peculiar feature of the meter, and is shown clearly to an enlarged scale by Fig. 2.



DESCRIPTION OF PRESSURE CHAMBER.

It consists of three separate pieces, the outer one, A, of which couples the parts B, C, together, leaving a continuous opening, D, around the bore, which may be of any required width. The part B is recessed to form a pressure chamber connected to the guage by an opening, E. The parts B and C are faced, so that when drawn together by the coupling A they form a watertight joint at F, and the ends of cones are screwed into corresponding recesses in B and C.

This form of throat possesses several advantages. The continuous opening gives a more accurate value of the pressure than a series of holes; it can be faced without any burr, and its width can be adjusted with accuracy. Also, the diameter of the throat at the point where the pressure is measured can be determined with the greatest possible accuracy, and, moreover, is open to inspection.

The parts can be rotated relatively to the others without destroying the joint, an advantage in setting up the apparatus.

It should be pointed out that the measurement of the pressure at the down stream end of the cone is not essential for practical measurements; it was merely used for determining the total loss of head in the instrument. The down-stream pressure chamber can, therefore, be dispensed with in ordinary use.

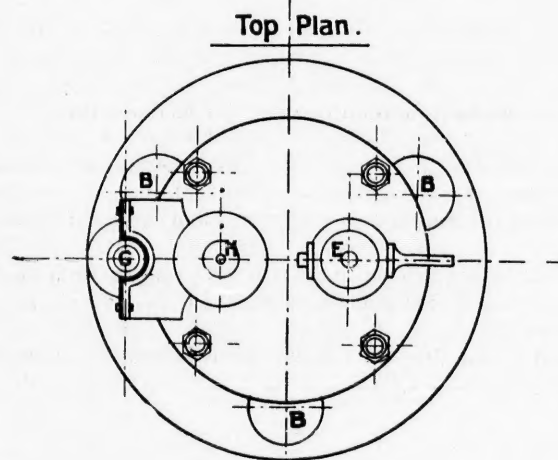
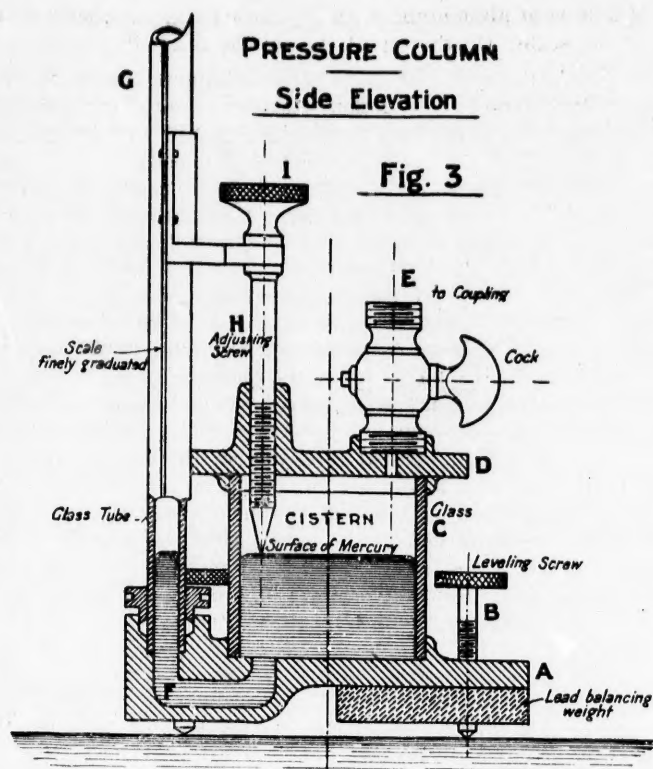
GAUGES.

The gauges used for measuring the pressure of the water in the pipes are of unusual pattern.

Fig. 3 is a section, in which A is an iron base plate about $\frac{3}{8}$ inch thick, provided with three brass levelling screws, B. On a ring of india-rubber on the plate stands a cylindrical glass vessel, C, or reservoir, about $2\frac{1}{2}$ inches in diameter, with an aluminium cover, D. Four rods, not shown, screwed into the base plate and passing through the cover, have screwed ends and nuts at the top, by which the reservoir is pressed down on the rubber ring. Near the centre of the aluminium top is a tap, E, and a nipple, into which is screwed a T piece, the upper part of which is provided with an air cock, while the horizontal branch is connected with the pressure chamber by means of a piece of rubber tubing.

Beneath the iron base is a pipe, F, connecting the reservoir with a vertical glass tube, G, of $\frac{3}{16}$ inch bore and 35 inches in length. Behind this tube is a scale reading up to 32 inches, and movable relatively to the tube and reservoir.

At the lower end of the scale a spindle, H, is attached to the back, and passes through a gland into the reservoir. The bottom of



this spindle is of aluminium, with a point on the same level as the zero of the scale. On the top of the spindle is a milled head, I, by means of which the spindle and scale can be moved up or down until the point reaches the surface of the mercury in the reservoir. The reading of the scale is then evidently the height of the mercury column above the level of the mercury in the reservoir. There is a small steel auxiliary scale, with .01 inch divisions, which measures the displacement, from a datum, of the zero of the main scale, when the latter is moved until the point touches the surface of the mercury. The zeros of the small auxiliary scales of the two gauges are set on the same level.

It is easily seen that the displacement of the main scale, or, rather, the difference of the displacements of the main scales from the same level, is a correction in pressure of water to be applied to the difference of the readings of the mercury columns. The readings of the mercury columns were made upon steel scales, graduated to .01 inches, and fixed with their zeros on the same level as the surface of the mercury in the glass tube, when the pointer touches the surface of the mercury in the reservoir before the water is admitted. In this way there is no error due to difference, in capillarity due to small differences of diameter in the glass tubes of the two gauges. Each gauge is read by means of a Marten's telescope, which is fixed up at about 6 feet from the gauge and adjusted to the level of the meniscus. With a good light and steady pressure the readings can be taken to .001 inch.

For measuring the vacuum at the throat, the pressure chamber was connected to a long glass tube running up parallel to the tube of the pressure gauge, to which it was connected at the top, where it was provided with an air tap. One of the difficulties to be contended with is leakage of air into the vacuum, and several trial runs were made to detect and remedy this leakage before commencing accurate work. All pipe joints were covered with tallow and wrapped with telegrapher's tape, and, it is believed, there is no error from this source.

GENERAL DESCRIPTION OF THE APPARATUS.

The water was drawn from the experimental tank in the hydraulic laboratory, and passed by a 1½ inch pipe, 12 feet long, to the Venturi; the outflow pipe was 6 feet long and of the same diameter.

The tank was provided with means for setting and regulating the head, and no difficulty was experienced in keeping the head practically constant.

Instead of the pipe discharging under water, an artificial head was created by screwing a nozzle of ¾ inch diameter on the end of

the outflow pipe. The function of this was to ensure that the pipe ran full below the Venturi.

Through this nozzle the water discharged into a bifurcated shoot supported on a wooden frame and pivoted in such a manner that either leg could be brought in front of the nozzle, the instant of change being recorded on a chronograph. One leg discharged to waste and the other to the measuring tank.

The tank, one of a series of five, was beneath the floor level, and was 9 feet by 6 feet by 3 feet 6 inches, cast iron-encased in concrete, and experiment showed that there was no leakage whatever.

To the tank was connected a 4 inch vertical brass pipe forming a float chamber. The float was attached to a vertical $\frac{3}{16}$ inch brass rod with a pointer at the upper end indicating on a brass scale the quantity of water in the tank. A fine cord fastened to the top of the rod rose vertically, passed over a frictionless pulley and carried a balance weight, which kept the cord taut and prevented the pointer from rubbing against the scale.

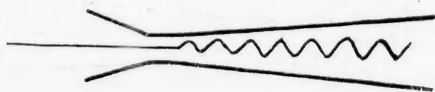
SETTING OF GAUGES.

The gauges stood on a firm slab of slate, and were accurately levelled. The zeros of the auxiliary scales were set on the same level, and referred to the scale on the supply tank, and also to the axis of the Venturi, by means of a Dumpy level.

THE CHARACTER OF THE MOTION.

In order to study the motion by aid of colour bands, a glass pipe, Fig. 4, was drawn out to form a Venturi meter of approximately one-half the linear size of the meter under experiment.

Fig. 4



With a head of about three feet of water upon this meter a colour band was introduced, and the motion studied by its aid.

The water was always found to be stable in the up-stream cone as long as the water coming to it was stable; whereas, after passing the neck it immediately broke down into eddies as shown. On increasing the head the coloured band in the up-stream cone still remained clearly defined until swept out by the water breaking down in the supply pipe above.

It, therefore, appears that the motion in the up-stream cone may be taken as stable until the limit for the up-stream pipe is reached,

and this limit, for a pipe of uniform base, has been shown by Osborne Reynolds to be given by the formula:—

$$V_c = 0.039 \frac{f(T)}{D}$$

where

V_c = critical velocity.

$$f(T) = (1 + .0336 T + .000221 T^2)^{-1}$$

T = temperature centigrade.

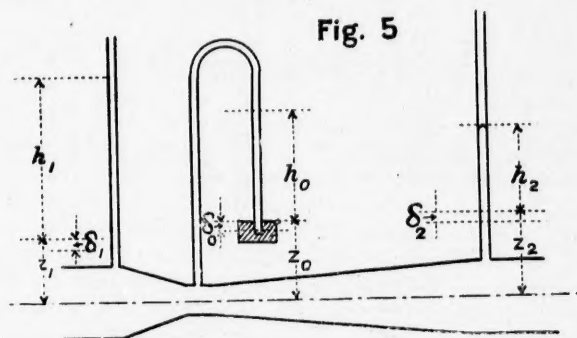
D = diameter of the pipe in inches.

In assuming Bernoulli's Theorem we are making the assumption that the fluid is frictionless, or else that the motion is such that the loss due to fluid friction may be disregarded. In the present case the loss in friction is large, being given by the values P_1, P_2 Table I, and shown for different discharges in the curve Fig. 10.

This loss will be divided unequally between the two cones.

The loss in the up-stream cone will be much smaller than in the down-stream, partly because of its short length, but principally because the flow is in general stable, while in the down-stream cone the divergence of the walls causes eddy motion.

NOTATION.



Let Q = total quantity in gallons.

q = discharge in cubic feet per second.

h_1, h_0, h_2 = the readings of the gauges.

$\delta_1, \delta_0, \delta_2$ = the readings on auxiliary scales.

z_1, z_0, z_2 = heights of the zeros of the auxiliary scales above the axis of the Venturi

T = total time of run in seconds

τ = temperature of water.

Π_0 pressure of atmosphere.

p_1, p_0, p_2 = pressures at the pressure chambers in feet of water.

a_1, a_0, a_2 = areas of the throats.

v_1, v_0, v_2 = velocities at the throats.

C = coefficient of Venturi.

g = 32.176 for Montreal.

c = density of mercury.

If we can assume that Bernoulli's Theorem holds true for the motion in the Venturi the theory is extremely simple, and a simple expression can be obtained for the discharge.

Taking the axis of the pipe as level, and in the datum plane, we have:--

$$\frac{p_1}{\omega} + \frac{v_1^2}{2g} = \frac{p_0}{\omega} + \frac{v_0^2}{2g}$$

$$\text{Now } \frac{p_1}{\omega} = \frac{\Pi_0}{\omega} + \frac{\sigma h_1 + \delta_1 + z_1}{12}$$

$$\frac{p_0}{\omega} = \frac{\Pi_0}{\omega} - \frac{\sigma h_0}{12}$$

$$\therefore \frac{p_1 - p_0}{\omega} = \frac{\sigma (h_1 + h_0)}{12} + \frac{\delta_1 + z_1}{12} = H \text{ say}$$

$$\therefore v_0^2 - v_1^2 = 2g H$$

$$\text{Now } v_1 = \frac{a_0}{a_1} v_0$$

$$\therefore v_0^2 - \frac{a_0^2}{a_1^2} v_0^2 = 2g H$$

$$v_0^2 = \frac{a_1^2}{a_1^2 - a_0^2} 2g H$$

$$q = a_0 v_0 = \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \sqrt{2g H}$$

To allow for friction, etc., we may put

$$q = c \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \sqrt{2g H}$$

LIMITS OF ERROR.

Total discharge, Q.—In the gauging tank one gallon corresponded to .087 inch, and the position of the pointer could be read to .005 inch, or about .06 of a gallon. Q varied from 64 to 336 gallons, so that the error of reading varied from $\frac{1}{1024}$ to $\frac{1}{5120}$.

As Q was measured by differences, the greatest probable error would be twice the above. All the readings were taken independently by two observers.

Time.—The chronograph was operated from a standard clock in the laboratory, the clock being compared with the McGill Observatory clock each day. The distance on the record corresponding to 1 second was 0.4 inches, and readings could be made to .005 inches, so that the greatest possible error in a reading was $\frac{1}{80}$ seconds, or the combined error of the readings at start and finish was $\frac{1}{40}$ seconds.

The total time was always about 1,200 seconds, so that the error would not exceed 1 in 100,000.

The only point to be noted in connection with the chronograph was the use of two pens, or, rather, one pen and a glass stylus following in its track, the former leaving enough ink for the stylus. These two pens recorded in opposite directions, so that there could be no confusion between the seconds' mark and the mark recording the instant of throwing over the shoot.

MEASUREMENT OF DIAMETERS OF VENTURI.

This was done on a dividing machine reading to .0001 inches; the diameter of the throat was 0.3798 inches; area = .000787 sq. ft.; the diameter of the up-stream throat was 1.627 inches and the diameter of the down-stream throat was 1.626 inches.

Mean area = .014411 square feet.

MEASUREMENT OF PRESSURES.

In all except runs 75 to 85 inclusive, mercury was used in the pressure and vacuum gauges, and readings were taken to .001 in.

The accuracy with which pressures are obtained depends not only on the reading of the gauge, but on the settings of the zeros and of the pointers. The levelling of the gauges could be done to within .01 inches of water, or say, .001 inches of mercury. The error of setting the pointer would not probably be more than .002 inches, so that the combined error of any single reading is not likely to be more than .005 inches. As the pressure readings were generally very constant throughout a run, the probable error of the mean would not be as large as this, and the error of $P_1 - P_2$ is probably not more than .003 inches, since ten readings were taken for each run. The vacuum, however, generally varied a good deal, and an error of .01 inches in the reading of $P_0 + P_1$ is very probable. The greatest value of $P_1 - P_2$ is about 23 inches of mercury, giving an error of, say, 1 in 8,000. The least reading for mercury was about .72 inch, giving an error of 1 in 240.

The greatest value of $P_1 + P_0$ was about 50 inches, giving an error of, say, 1 in 5,000, and the least about 1.2 inches, giving 1 in 120.

In the few experiments in which water was used in the gauges instead of mercury, the readings were only taken to the nearest $\frac{1}{100}$ of an inch, and the order of the error is about the same as given above.

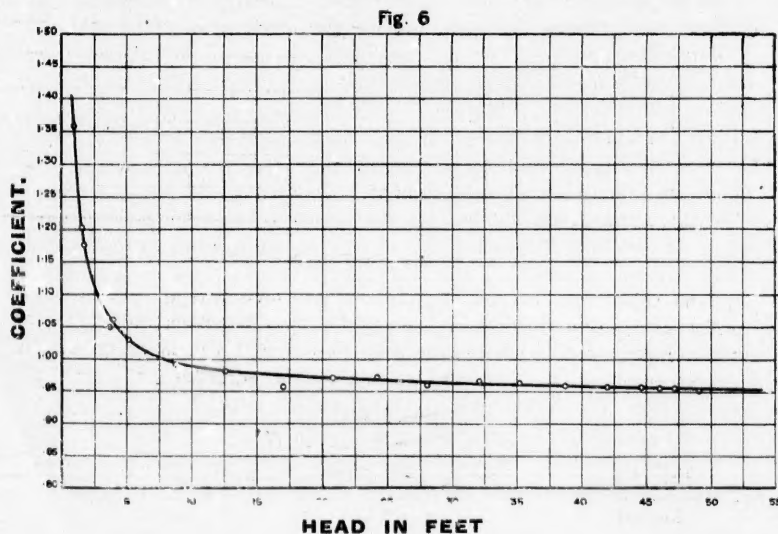
METHOD OF EXPERIMENT.

As a general rule three runs were made at each head, in a few cases four. The water was always turned on and allowed to flow to waste for half-an-hour or longer before commencing a series of runs, as it was found that the vacuum did not reach a steady condition until some time after the water was turned on. Stop watches were employed as a rough check on the chronograph and for determining time of readings.

DISCUSSION OF THE OBSERVATIONS.

The observations are given on Table I, and the value of C has been deduced from the mean values of each set of experiments, on the assumption that Bernouilli's law holds for the cone.

It will be noticed that c is in general less than unity, and is least for the highest values of H gradually increasing with the diminution of head until a head of about 8 feet is reached, when it passes through the value unity. With still lower heads the increase in c is much more marked, rising to a value of 1.358 for a head of 0.972 feet.



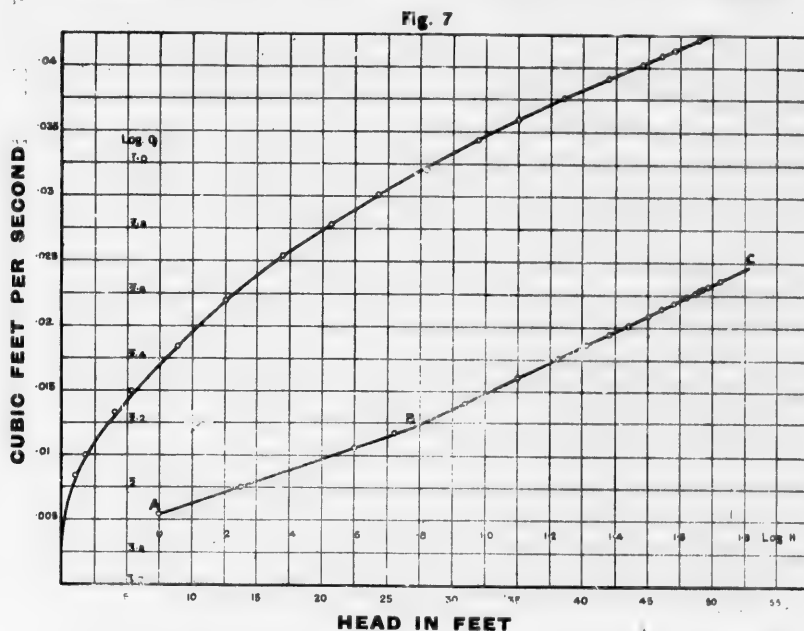
The relation of c to H is plotted in Fig. 6, and the wide variation in the value of c is clearly apparent.

It is noticeable that Herschel* found much lower values of c , but none of his experiments give such a high value as 1.358.

For moderate heads the experiments bear out the usual assumption, for this form of meter, that the constant does not differ much from unity.

For low heads, however, this does not hold, and a comparatively large error is introduced by assuming a coefficient of unity.

The variation in the value of c appears to have an intimate connection with the question of the stability of flow in the up-stream cone, and on plotting the discharges as ordinates with the heads as abscissae, Fig. 7,



It was seen that the upper part of the curve was of a somewhat different character to the lower part. This becomes more evident when logarithmic co-ordinates are used, for if the law concerning Q and H be taken to be represented by the equation

$$q = k H^n$$

$$\log q = \log k + n \log H$$

and the slope of the line gives the value of n .

* loc cit.

This has been done in Fig. 7, in which the line A B has a slope whose tangent is approximately 0.341, while B C has an inclination whose tangent is 0.478. It, therefore, appears that in this meter for low velocities the discharge is proportional to a root of H higher than the square and less than the cube, while for higher velocities it varies nearly as the square root of the head.

This result affords a clue to the rise in the value of c for low heads, for since its value has been deduced from the formula

$$q = k H^{\frac{1}{2}}$$

k being some constant, while the law for the lower part of the scale is represented by

$$q = k^1 H^n$$

where

$$n > 2 < 3$$

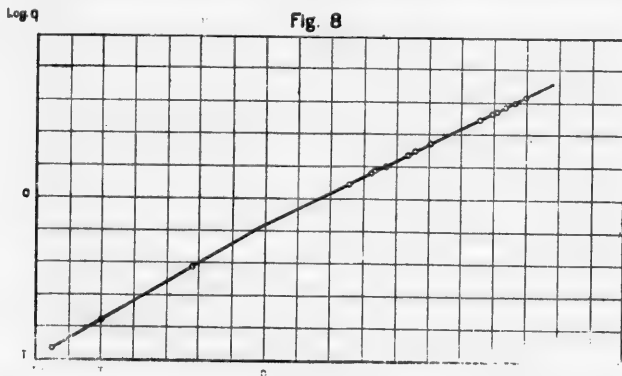
it is clear that the value of c , being a factor of k , will necessarily increase.

It is instructive to compare these results with those obtained by Herschel on a meter for a pipe of one foot diameter and on one for a nine foot pipe.

These results are tabulated in his paper, and from his Table I the values of q and H have been plotted logarithmically.

For various reasons, which are fully stated in the paper, a number of the results were of doubtful accuracy, and, in consequence, are not considered here.

The most reliable results appear to be those numbered 37 to 60, and these were plotted logarithmically, and are shown on Fig. 8.

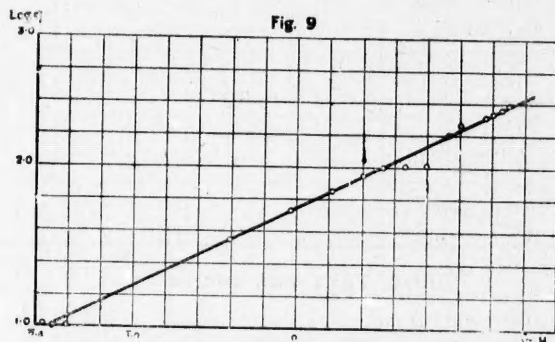


In this case, the value of n for the higher velocities came out as .49, while the low velocities of experiments 58, 59 and 60 gave a value of $n = 0.6$.

The reason for this discrepancy is not clear, but it may be pointed out that this first series of experiments was conducted under great disadvantages, in spite of the care taken to ensure accurate results.

The second set of experiments was made upon a meter for a pipe 9 feet in diameter, all the linear dimensions being approximately 9 times greater than those of the first meter.

The observations were massed into groups, which an inspection of Table II. shows to be justifiable, and the mean results plotted in Fig. 9.



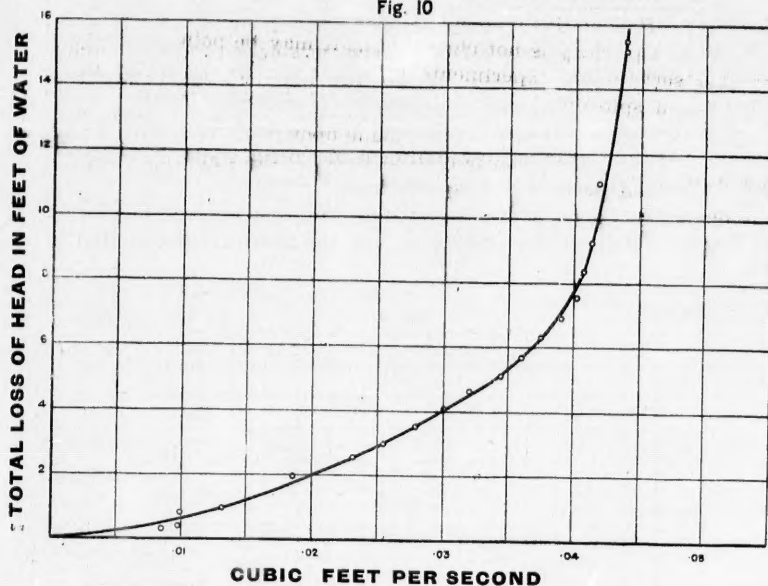
All the points were found to lie on a straight line, having an inclination $\tan^{-1} \frac{1}{2}$ almost exactly, and, therefore, verifying the theoretical law in the case of large meters.

These results point to the conclusion that in large Venturi meters the discharge is very approximately proportional to the square root of the head throughout the whole range, while in small meters the discharge does not apparently follow this law for low heads, but does so approximately for high heads.

It appears, therefore, that a small meter would require special calibration for use when discharging small quantities of water, since it does not follow the square root law, and this renders the meter unsuitable for use in the measurement of, say, a domestic supply, where it is important that small quantities should be accurately measured, as well as losses due to leakage in pipes, defective taps and the like. Moreover, there is a considerable loss of head in such small meters, as can be seen from the Table, Fig. 10, and this is a further disadvantage.

On the other hand, its great advantage for the measurement of large quantities of water is manifest, in fact, it is the only practicable method of measuring the water passing through a main, and it has been shown repeatedly that the loss of head in this case is small.

Fig. 10



No. of Expt.	HEADS IN FEET OF WATER.			Total time seconds.	Total quantity gallons.	Cubic feet per sec.	Coeffct. C
	P_1 Up stream gauge.	P_0 Centre vacuum gauge.	P_2 Down stream gauge.				
18	16.120	32.898	4.918	1,205.3	313.95	.0418	.9494
19	16.122	32.886	4.918	1,205.0	314.02	.0418	
20	16.128	32.910	4.924	1,205.3	314.76	.0419	
21	14.142	33.098	4.838	1,205.6	310.46	.0413	.9578
22	14.138	33.047	4.841	1,204.7	311.09	.0414	
23	14.141	33.046	4.839	1,205.3	311.36	.0415	
23b	13.174	33.062	4.706	1,205.0	306.44	.0408	.9567
24	13.161	32.992	4.708	1,205.0	307.36	.0409	
25	13.160	32.987	4.714	1,205.5	307.60	.0409	
25a	12.175	32.546	4.568	1,205.2	301.84	.0402	.9564
26	12.176	32.487	4.563	1,205.4	302.47	.0401	
27	12.174	32.490	4.565	1,205.3	303.85	.0403	
28	11.224	30.776	4.272	1,205.2	292.44	.0389	.9566
29	11.224	30.781	4.375	1,205.4	293.36	.0391	
30	11.224	30.785	4.281	1,205.7	292.90	.0390	
31	10.314	28.266	3.896	1,205.7	280.12	.0373	.9569
32	10.306	28.239	3.921	1,205.2	281.49	.0375	
33	10.302	28.228	3.920	1,206.0	281.28	.0374	

No. of Expt.	Head in feet of water.			Total time seconds.	Total quantity gallons.	Cubic feet per sec.	Coeff C.
	P_1 Up stream guage.	P_0 Centre vacuum guage.	P_2 Down stream guage.				
34	9.362	25.813	3.586	1,205.0	262.00	.0358	0.9619
35	9.360	25.791	3.587	1,204.9	269.75	.0359	
36	9.360	25.767	3.587	1,205.2	269.79	.0359	
37	8.440	23.519	3.220	1,206.0	255.27	.0340	0.9644
38	8.424	23.696	3.267	1,206.3	257.83	.0343	
39	8.420	23.527	3.263	1,206.3	257.93	.0343	
40	8.417	23.438	3.258	1,206.3	257.93	.0343	
41	7.516	20.525	2.821	1,205.2	239.83	.0319	0.9587
42	7.509	20.527	2.820	1,035.6	215.98	.0319	
43	7.514	20.544	2.825	1,206.5	240.40	.0319	
44	6.549	17.726	2.462	1,205.9	227.46	.0303	0.9719
45	6.566	17.626	2.459	1,206.1	225.46	.0300	
46	6.571	17.629	2.456	1,206.12	225.72	.0300	
47	5.637	15.154	2.093	1,206.5	209.89	.0278	0.9704
48	5.637	15.165	2.093	1,206.6	208.85	.0278	
49	5.636	15.146	2.091	1,206.3	209.66	.0279	
50	5.636	15.138	2.091	1,205.7	209.54	.0279	
51	4.709	12.325	1.709	1,207.4	191.72	.0255	0.9572
52	4.709	12.223	1.707	1,207.1	191.09	.0254	
53	4.709	12.138	1.701	1,205.4	189.44	.0252	
54	3.792	8.955	1.257	1,206.0	165.63	.0220	0.9800
55	3.792	8.916	1.257	1,205.8	165.52	.0220	
56	3.791	8.879	1.254	1,205.9	165.29	.0220	
57	2.873	5.879	0.845	1,206.4	138.85	.0185	0.9929
58	2.870	5.853	0.843	1,205.9	134.59	.0184	
59	2.868	5.843	0.844	1,205.5	138.62	.0185	
60	1.943	3.382	.506	1,204.9	111.70	.0149	1.0296
61	1.943	3.324	.501	1,205.3	111.32	.0148	
62	1.944	3.288	.498	1,205.2	110.97	.0149	
63	1.003	1.012	.116	1,105.2	74.46	.0099	1.1757
64	1.001	.727	.127	1,205.4	74.23	.0099	
65	0.998	.759	.131	1,205.9	74.00	.0098	
66	20.956	33.183	5.676	1,205.8	335.00	.0447	0.9537
67	20.959	33.197	5.681	1,206.1	335.90	.0447	
68	20.9606	33.079	5.684	1,206.0	336.15	.0447	
71	11.471	2.524	0.375	1,206.2	99.89	.0133	1.0596
72	1.474	2.494	0.380	1,206.1	99.89	.0133	
73	1.479	2.438	0.370	1,206.9	99.32	.0132	

No. of Expt.	HEAD IN FEET OF WATER.			Total time seconds.	Total quantity gallons.	Cubic feet per sec.	Coefft. C.
	P_1 Up stream gauge.	P_0 Centre vacuum gauge.	P_2 Down stream gauge.				
75	.941	.880	.552	1,206.9	75 12	.0610	1.2021
76	.945	.782	.579	1,205.2	74.93	.0010	
77	.944	.677	.570	1,208.2	72.63	.0096	
78	.944	.642	.569	1,206.2	72.08	.0096	
79	.944	.634	.567	1,207.1	72.30	.0096	
80	.775	.207	.459	1,207.2	63.39	.0084	1.3583
81	.774	.195	.457	1,207.2	63.33	.0084	
82	.773	.195	.455	1,206.0	63.28	.0084	
83	1.485	2.314	.538	1,205.85	96.26	.0128	1.0491
84	1.571	2.228	.626	1,205.80	96.72	.0129	
85	1.570	2.204	.664	1,206.55	96.72	.0129	

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